

Jan 2012

6. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 \leq x < 1 \\ x - \frac{1}{2} & 1 \leq x \leq k \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

- (a) Sketch the graph of $f(x)$. (2)
- (b) Show that $k = \frac{1}{2}(1 + \sqrt{5})$. (4)
- (c) Define fully the cumulative distribution function $F(x)$. (6)
- (d) Find $P(0.5 < X < 1.5)$. (2)
- (e) Write down the median of X and the mode of X . (2)
- (f) Describe the skewness of the distribution of X . Give a reason for your answer. (2)

3.

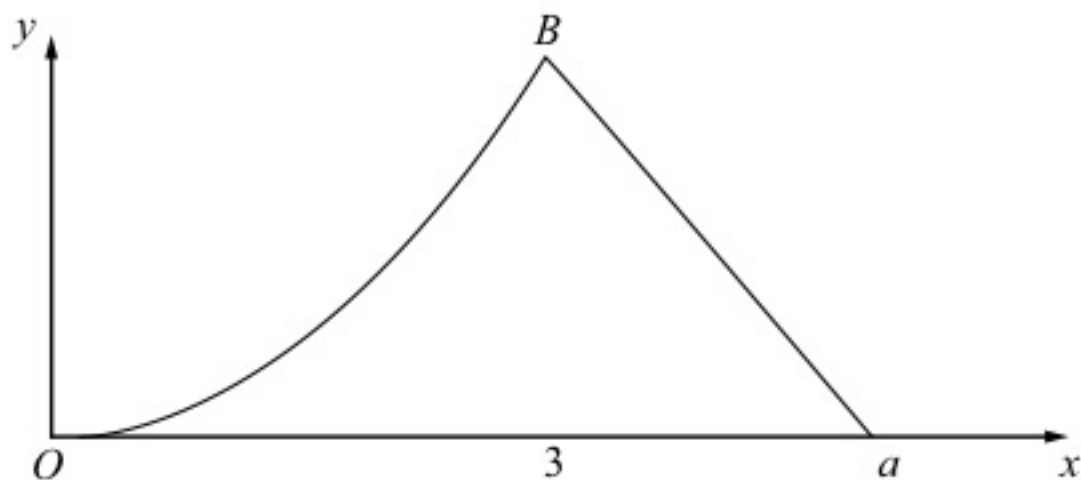
**Figure 1**

Figure 1 shows a sketch of the probability density function $f(x)$ of the random variable X .

For $0 \leq x \leq 3$, $f(x)$ is represented by a curve OB with equation $f(x) = kx^2$, where k is a constant.

For $3 \leq x \leq a$, where a is a constant, $f(x)$ is represented by a straight line passing through B and the point $(a, 0)$.

For all other values of x , $f(x) = 0$.

Given that the mode of $X =$ the median of X , find

(a) the mode, (1)

(b) the value of k , (4)

(c) the value of a . (3)

Without calculating $E(X)$ and with reference to the skewness of the distribution

(d) state, giving your reason, whether $E(X) < 3$, $E(X) = 3$ or $E(X) > 3$. (2)

5. A continuous random variable X has the probability density function $f(x)$ shown in Figure 1.

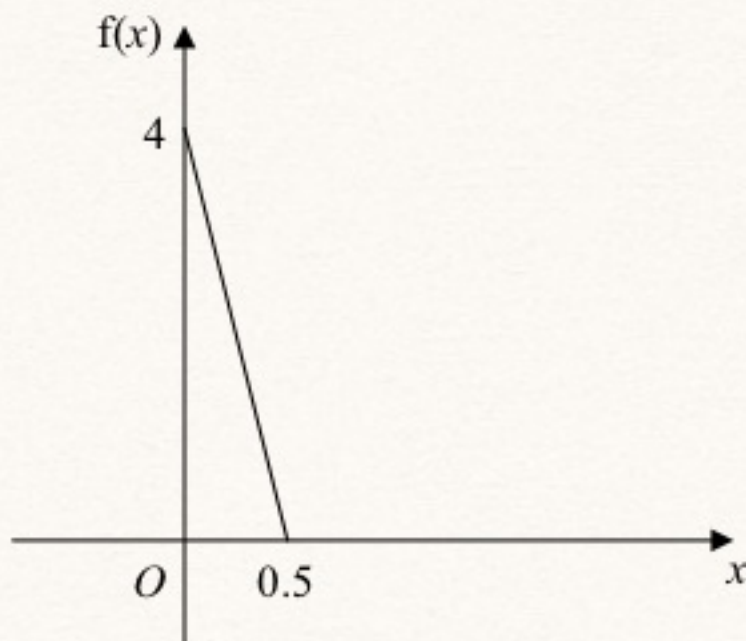


Figure 1

- (a) Show that $f(x) = 4 - 8x$ for $0 \leq x \leq 0.5$ and specify $f(x)$ for all real values of x . **(4)**
- (b) Find the cumulative distribution function $F(x)$. **(4)**
- (c) Find the median of X . **(3)**
- (d) Write down the mode of X . **(1)**
- (e) State, with a reason, the skewness of X . **(1)**

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4. The lifetime, X , in tens of hours, of a battery has a cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{4}{9}(x^2 + 2x - 3) & 1 \leq x \leq 1.5 \\ 1 & x > 1.5 \end{cases}$$

- (a) Find the median of X , giving your answer to 3 significant figures. (3)
- (b) Find, in full, the probability density function of the random variable X . (3)
- (c) Find $P(X \geq 1.2)$ (2)

A camping lantern runs on 4 batteries, all of which must be working. Four new batteries are put into the lantern.

- (d) Find the probability that the lantern will still be working after 12 hours. (2)

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2. A continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < -2 \\ \frac{x+2}{6}, & -2 \leq x \leq 4 \\ 1, & x > 4 \end{cases}$$

- (a) Find $P(X < 0)$.
- (b) Find the probability density function $f(x)$ of X .
- (c) Write down the name of the distribution of X .
- (d) Find the mean and the variance of X .
- (e) Write down the value of $P(X = 1)$.

7.

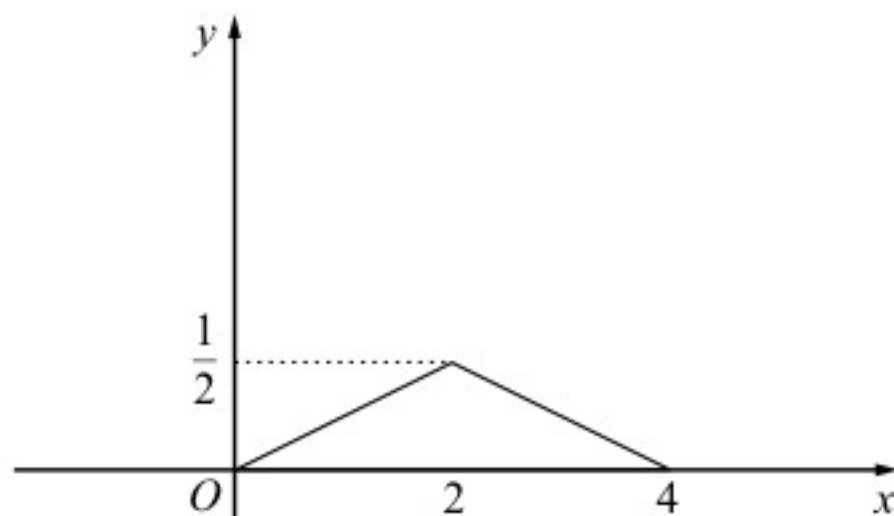


Figure 1

Figure 1 shows a sketch of the probability density function $f(x)$ of the random variable X . The part of the sketch from $x = 0$ to $x = 4$ consists of an isosceles triangle with maximum at $(2, 0.5)$.

(a) Write down $E(X)$.

(1)

The probability density function $f(x)$ can be written in the following form.

$$f(x) = \begin{cases} ax & 0 \leq x < 2 \\ b - ax & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(b) Find the values of the constants a and b .

(2)

(c) Show that σ , the standard deviation of X , is 0.816 to 3 decimal places.

(7)

(d) Find the lower quartile of X .

(3)

(e) State, giving a reason, whether $P(2 - \sigma < X < 2 + \sigma)$ is more or less than 0.5

(2)

7. A random variable X has probability density function given by

$$f(x) = \begin{cases} -\frac{2}{9}x + \frac{8}{9} & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the cumulative distribution function $F(x)$ can be written in the form $ax^2 + bx + c$, for $1 \leq x \leq 4$ where a , b and c are constants.

(3)

- (b) Define fully the cumulative distribution function $F(x)$.

(2)

- (c) Show that the upper quartile of X is 2.5 and find the lower quartile.

(6)

Given that the median of X is 1.88

- (d) describe the skewness of the distribution. Give a reason for your answer.

(2)

7. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leq x < 1 \\ kx^3 & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{5}$ (4)

(b) Calculate the mean of X . (4)

(c) Specify fully the cumulative distribution function $F(x)$. (7)

(d) Find the median of X . (3)

(e) Comment on the skewness of the distribution of X . (2)