C_1 Test MSemar (14.10.11).

Correction.

Question 1:

a- $3^x = 9^{y-1}$

show $x = 2y-2$.

$$3^x = (3^2)^{y-1}$$
$$3^x = 3^{2(y-1)} \iff 3^x = 3^{2y-2}$$
$$\therefore \quad x = 2y-2.$$

b- Solve $\begin{cases} x = 2y-2 & (1) \\ x^2 = y^2+7 & (2) \end{cases}$

$$\begin{cases} x = 2y-2 \\ (2y-2)^2 = y^2+7 \end{cases} \Rightarrow \begin{cases} x = 2y-2 \\ 4y^2+4-8y = y^2+7 \end{cases}$$

$$\begin{cases} x = 2y-2 \\ 4y^2+4-8y-y^2-7 = 0 \end{cases} \Rightarrow \begin{cases} x = 2y-2 \\ 3y^2-8y-3 = 0 \end{cases}$$

Solving $3y^2-8y-3=0$.

$3y^2-9y+y-3=0$.

$3y(y-3)+(y-3)=0$

$(y-3)(3y+1)=0$.

$\therefore y=3$ or $y=-1/3$.

$\begin{cases} y=3 & \text{gives } x = 2\times 3-2 = 4. \\ y=-1/3 & \text{gives } x = -2/3-2 = -8/3 \end{cases}$

Solutions of simultaneous are: $\begin{cases} x=4 & y=3 \\ x=-8/3 & y=-1/3 \end{cases}$ (or)

Question 2 :- $\begin{cases} x - 2y = 1 \\ 3xy - y^2 = 8 \end{cases} \Rightarrow \begin{cases} x = 2y + 1 \\ 3xy - y^2 = 8 \end{cases}$.

$\begin{cases} x = 2y+1 \\ 3(2y+1)y - y^2 = 8 \end{cases} \Rightarrow \begin{cases} x = 2y+1 \\ 6y^2 + 3y - y^2 - 8 = 0 \end{cases}$.

$\begin{cases} x = 2y+1 \\ 5y^2 + 3y - 8 = 0 \end{cases}$.

$\therefore$ Simultaneous produces $5y^2 + 3y - 8 = 0$.

Solving $5y^2 + 3y - 8 = 0$

$\begin{cases} ? + ? = +3 \\ ? \times ? = -40 \end{cases}$

8 and -5

$5y^2 + 8y - 5y - 8 = 0$

$5y^2 - 5y + 8y - 8 = 0$

$5y(y-1) + 8(y-1) = 0$

$(y-1)(5y+8) = 0$

$y = 1$ or $y = -8/5$.

When $y = 1$ $\quad x = 2 \times 1 + 1 = 3$ $\quad x = 3$ & $y = 1$.

When $y = -\frac{8}{5}$ $\quad x = 2 \times -8/5 + 1 = -\frac{16}{5} + 1 = \frac{-11}{5}$ $\quad x = -\frac{11}{5}$ & $y = -8/5$.

$\therefore$ Solutions to simultaneous are: $\begin{cases} x = 3 \text{ \& } y = 1 \\ x = -11/5 \text{ \& } y = -8/5 \end{cases}$

Question 3

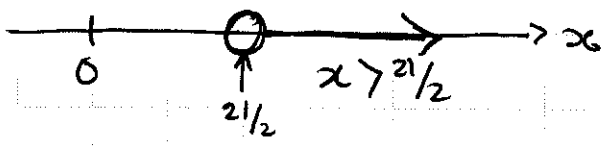
a) $3x - 8 > x + 13$

$2x - 8 > 13$

$2x > 13 + 8$

$2x > 21$.

$x > 21/2$.



b) $x^2 - 5x - 14 > 0$.

$? + ? = -5$ $\quad -7 ; +2$

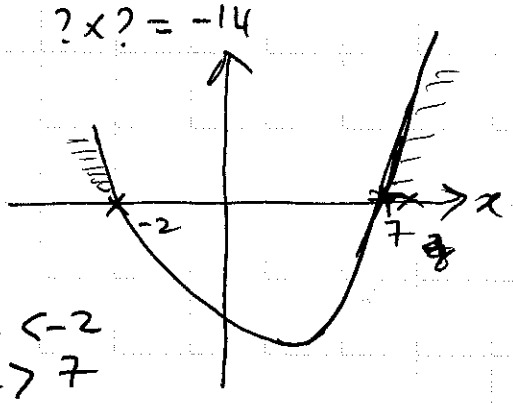
$? \times ? = -14$

$x^2 - 7x + 2x - 14 > 0$.

$x^2 + 2x - 7x - 14 > 0$.

$(x - 7)(x + 2) > 0$.

$x^2 - 5x - 14 > 0$ for $\begin{cases} x < -2 \\ x > 7 \end{cases}$



Question 4:

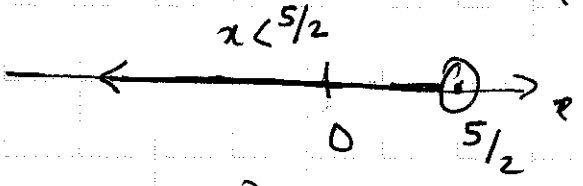
a) $6x - 7 < 2x + 3$

$6x - 2x - 7 < 3 + 2x - 2x$

$4x - 7 < 3$

$4x < 10$

$x < \frac{10}{4}$ or $x < \frac{5}{2}$.



b) $2x^2 - 11x + 5 < 0$

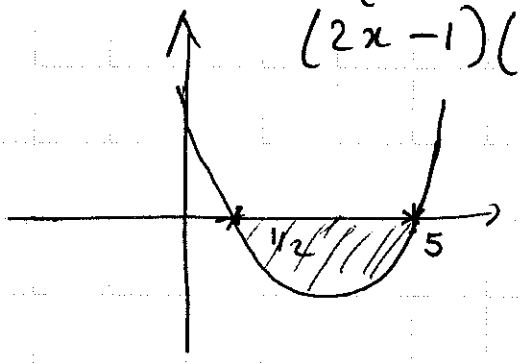
$2x^2 - x - 10x + 5 < 0$.

$x(2x - 1) - 5(2x - 1) < 0$.

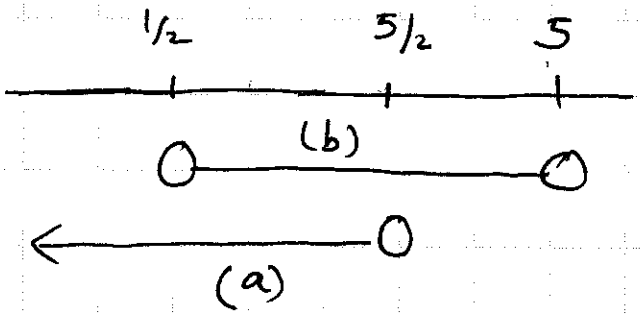
$(2x - 1)(x - 5) < 0$

$\Downarrow$

$1/2 < x < 5$.



c) $\begin{cases} 6x - 7 < 2x + 3 & (a) \\ \& \\ 2x^2 - 11x + 5 < 0 & (b) \end{cases}$



Set of values for both: $1/2 < x < 5/2$

Question 6: a) $3x^2 + 12x + 5 = p(x+q)^2 + r$.

p ?
q ?
r ?

$3x^2 + 12x + 5 = [3(x^2 + 4x) + 5]$.
$= 3[(x+2)^2 - 4] + 5$.
$= 3(x+2)^2 - 12 + 5$
$= 3(x+2)^2 - 7$.

$\begin{cases} p = 3 \\ q = +2 \\ r = -7 \end{cases}$.

b) Solve $3x^2 + 12x + 5 = 0 \iff$
$3(x+2)^2 - 7 = 0$
$(x+2)^2 = 7/3$
$x + 2 = \pm\sqrt{7/3} \Rightarrow$ $\begin{cases} x_1 = -2 + \sqrt{7/3} \\ x_2 = -2 - \sqrt{7/3} \end{cases}$

Question 7:

a - Solve $x^2+5x+2=0$

i) by completing the square:

$$\left(x+5/2\right)^2-\left(\frac{5}{2}\right)^2+2=0.$$

$$\left(x+5/2\right)^2=\left(\frac{5}{2}\right)^2-2.$$

$$\left(x+\frac{5}{2}\right)^2=\frac{25}{4}-\frac{8}{4}$$

$$\left(x+\frac{5}{2}\right)^2=\frac{17}{4} \Rightarrow x+\frac{5}{2}=\pm\sqrt{\frac{17}{4}}.$$

$$\begin{cases} x=-\frac{5}{2}+\frac{1}{2}\sqrt{17} \\ x=-\frac{5}{2}-\frac{1}{2}\sqrt{17} \end{cases} \text{ or } \begin{cases} x_1=\frac{-5+\sqrt{17}}{2} \\ x_2=\frac{-5-\sqrt{17}}{2} \end{cases}.$$

ii) Using formula: $x=\frac{-b+\sqrt{b^2-4ac}}{2a}$.

$$x=\frac{-5\mp\sqrt{25-4(2)}}{2} \Leftrightarrow x=\frac{-5\pm\sqrt{17}}{2}.$$

b) Solve $x^2-4x-3=0$.

i) Completing the square:

$$(x-2)^2+(2^2)-3=0.$$

$$(x-2)^2-4-3=0$$

$$(x-2)^2=7.$$

$$x-2=\pm\sqrt{7} \Rightarrow \begin{cases} x=2+\sqrt{7} \\ \text{or} \\ x=2-\sqrt{7} \end{cases}.$$

ii) Using formula:

$$x_{1,2}=\frac{4\mp\sqrt{16-4(-3)}}{2}=\frac{4\pm\sqrt{16+12}}{2}.$$

$$x_{1,2}=\frac{4\pm\sqrt{28}}{2}=2\pm\frac{1}{2}2\sqrt{7}=2\pm\sqrt{7}$$

Question 8:

a) sketch of $y = x^2 + 5x + 4$.

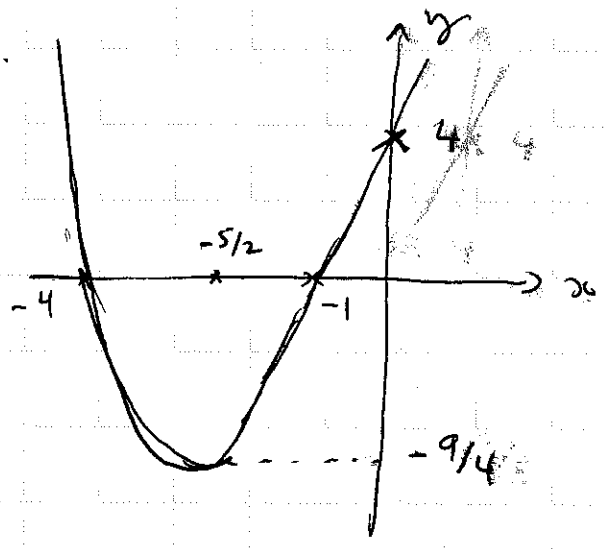
- X axis intercept: $x^2 + 5x + 4 = 0$, $(x+1)(x+4) = 0$. $\underline{x = -1}$ or $\underline{x = -4}$.
- Y axis intercept: $x = 0$ $y = 4$

- X axis intercept: $(x = -1 \quad y = 0)$, $(x = -4 \quad y = 0)$
- Y intercept: $(x = 0 \; ; y = 4)$

minimum ? $x^2 + 5x + 4 = \left(x + 5/2\right)^2 - \left(\frac{5}{2}\right)^2 + 4$.

$= \left(x + 5/2\right)^2 - \frac{9}{4}$.

• min $\left(-5/2 \; ; -9/4\right)$.



b) sketch $y = 2x^2 + x - 3$.

i) Intersection with Y axis: $x = 0$ $y = 2\times 0^2 + 0 - 3$

$y = -3$.

$(0; -3)$.

ii) Intersection with X axis:

$$2x^2 + x - 3 = 0$$
$$2x^2 - 2x + 3x - 3 = 0$$
$$2x(x-1) + 3(x-1) = 0$$
$$(x-1)(2x+3) = 0$$
$$x = 1 \quad \text{or} \quad x = -3/2.$$

with X axis: $(1; 0)$ & $(-3/2; 0)$.

iii) Minimum: completing the square.

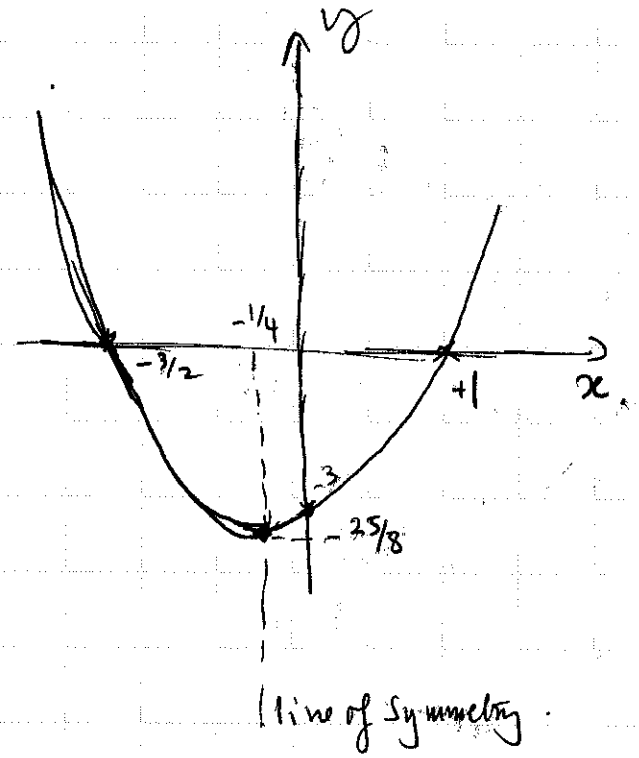
$$y = 2x^2 + x - 3 = 2\left(x^2 + \tfrac{1}{2}x\right) - 3$$

$$y = \left[2\left[\left(x + \tfrac{1}{4}\right)^2 - \tfrac{1}{16}\right] - 3\right.$$

$$y = 2\left(x + 1/4\right)^2 - \frac{1}{8} - 3$$

$$= 2\left(x + 1/4\right)^2 - \frac{25}{8}$$

Min $\left(-1/4; -25/8\right)$



Question 9: k ? $kx^2 + 8x + 5 = 0$ has real roots

this means $\Delta = b^2 - 4ac > 0$ (discriminant)

$\Delta = 64 - 4(k)(+5)$

$= 64 - 20k$.

$\Delta > 0 \iff 64 - 20k > 0$.

$64 > 20k$

or $20k < 64$

$k < \frac{64}{20}$.

$k < \frac{16}{5}$ or $k < 3\frac{1}{5}$.

For any given value of k / $k < 3\frac{1}{2}$,

$kx^2 + 8x + 5 = 0$ has real roots.