Centre No.						Paper Reference	Surname	Initial(s)
							Mrs OSMANE	
Candidate No.						6677/01	Signature	

Paper Reference(s)

6677/01

Edexcel GCE

Mechanics M1

Advanced/Advanced Subsidiary

Wednesday 18 May 2011 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. At time $t = 0$ a ball is projected vertically upwards from a point O and rises to a maximum height of 40 m above O. The ball is modelled as a particle moving freely under gravity.

(a) Show that the speed of projection is 28 m s^{-1}.

(3)

(b) Find the times, in seconds, when the ball is 33.6 m above O.

(5)

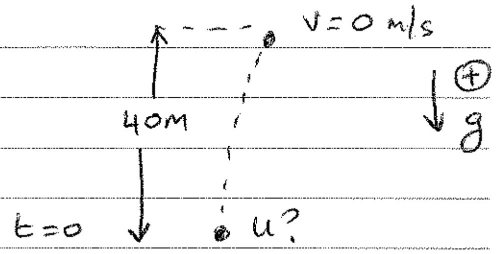
a) $s = 40m$

u ?

$v = 0 m/s$

$a = -9.8 m/s^2$

t



$v^2 - u^2 = 2as \Rightarrow u^2 = v^2 - 2as.$

$u = \sqrt{v^2 - 2as}.$

$u = \sqrt{0^2 - 2(-9.8)(40)}$

$\boxed{u = 28 m/s}$

b) $s = 33.6m$

$u = 28 m/s$

v

$a = -9.8 m/s^2$

t ?

$s = ut + \frac{1}{2}at^2$

$33.6 = 28t + \frac{1}{2}(-9.8)t^2 \Rightarrow 4.9t^2 - 28t + 33.6 = 0$

in the format $ax^2 + bx + c = 0$ $\quad a = 4.9$

$b = -28$

$c = 33.6.$

$t_1 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} = \dfrac{+28 - \sqrt{(-28)^2 - 4 \times 4.9 \times 33.6}}{2 \times 4.9} = 1.71 \text{ sec}$

$t_2 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} = \dfrac{+28 + \sqrt{(-28)^2 - 4 \times 4.9 \times 33.6}}{2 \times 4.9} = 4 \text{ sec}$

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Question 1 continued

Q1

(Total 8 marks)

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2. Particle P has mass 3 kg and particle Q has mass 2 kg. The particles are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision, P has speed 3 m s^{-1} and Q has speed 2 m s^{-1}. Immediately after the collision, both particles move in the same direction and the difference in their speeds is 1 m s^{-1}.

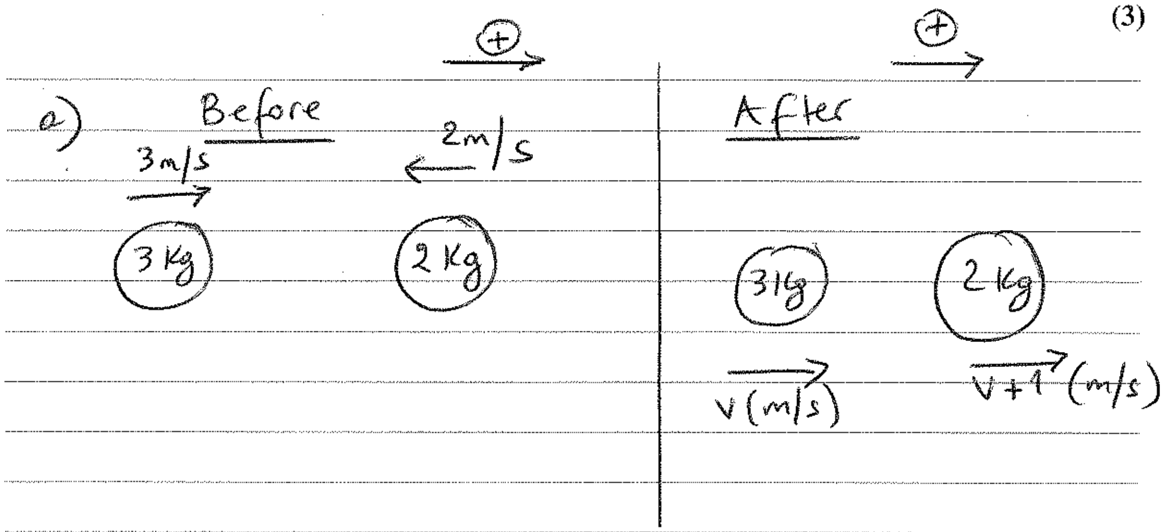
(a) Find the speed of each particle after the collision.

(5)

(b) Find the magnitude of the impulse exerted on P by Q.

(3)

a)



Conservation of Momentum:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$

$$3 \times 3 + 2 \times (-2) = 3v + 2(v+1).$$

$$9 - 4 = 5v + 2$$

$$v = \frac{3}{5}$$

Speed of P : $v = 0.6$ m/s.

Speed of Q : $v = 0.6 + 1 = 1.6$ m/s.

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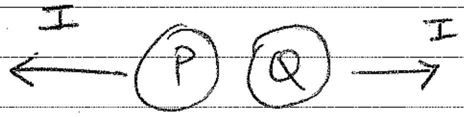
Question 2 continued

b)



Impulse exerted on P by Q.

$I = m(v - u)$.

$= 3(-0.6 - (-3))$

$= 3 \times 2.4$

$\boxed{I = 7.2 \text{ Ns}}$..

Q2

(Total 8 marks)

3.

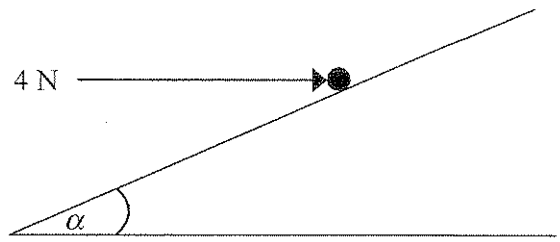


Figure 1

A particle of weight W newtons is held in equilibrium on a rough inclined plane by a horizontal force of magnitude 4 N. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 1.

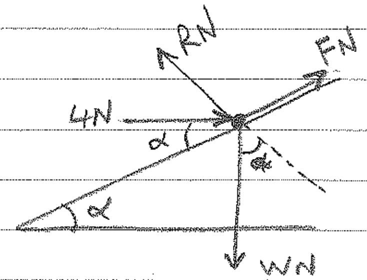
The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

Given that the particle is on the point of sliding down the plane,

(i) show that the magnitude of the normal reaction between the particle and the plane is 20 N,

(ii) find the value of W.

(9)



$F = \mu R$ (Friction force)

Particle held in equilibrium $\Rightarrow \Sigma F = 0$.

Vertically: $R = w\cos\alpha + 4\sin\alpha$ ①

Horizontally: $w\sin\alpha = 4\cos\alpha + \mu R$. ②

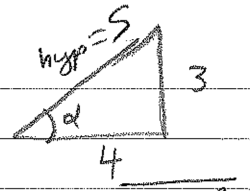
① into ② : $w\sin\alpha = 4\cos\alpha + \mu(w\cos\alpha + 4\sin\alpha)$

$w\sin\alpha = 4\cos\alpha + \mu w\cos\alpha + 4\mu\sin\alpha$.

$w(\sin\alpha - \mu\cos\alpha) = 4\cos\alpha + 4\mu\sin\alpha$.

$\boxed{w = \frac{4\cos\alpha + 4\mu\sin\alpha}{\sin\alpha - \mu\cos\alpha}}$.

Question 3 continued



$$\tan\alpha = \frac{3}{4}$$

$$hyp = \sqrt{3^2+4^2} = 5$$

$$\sin\alpha = \frac{3}{5} = 0.6$$

$$\cos\alpha = \frac{4}{5} = 0.8$$

$$w = \frac{4\times0.8 + 4\times\frac{1}{2}\times0.6}{0.6 - \frac{1}{2}\times0.8}$$

$$\boxed{w = 22N}$$

$$(1) \Rightarrow \quad R = 22\times0.8 + 4\times0.6$$

$$\boxed{R = 20N}$$

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Question 3 continued

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Question 3 continued

Q3

(Total 9 marks)

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4. A girl runs a 400 m race in a time of 84 s. In a model of this race, it is assumed that, starting from rest, she moves with constant acceleration for 4 s, reaching a speed of 5 m s^{-1}. She maintains this speed for 60 s and then moves with constant deceleration for 20 s, crossing the finishing line with a speed of V m s^{-1}.

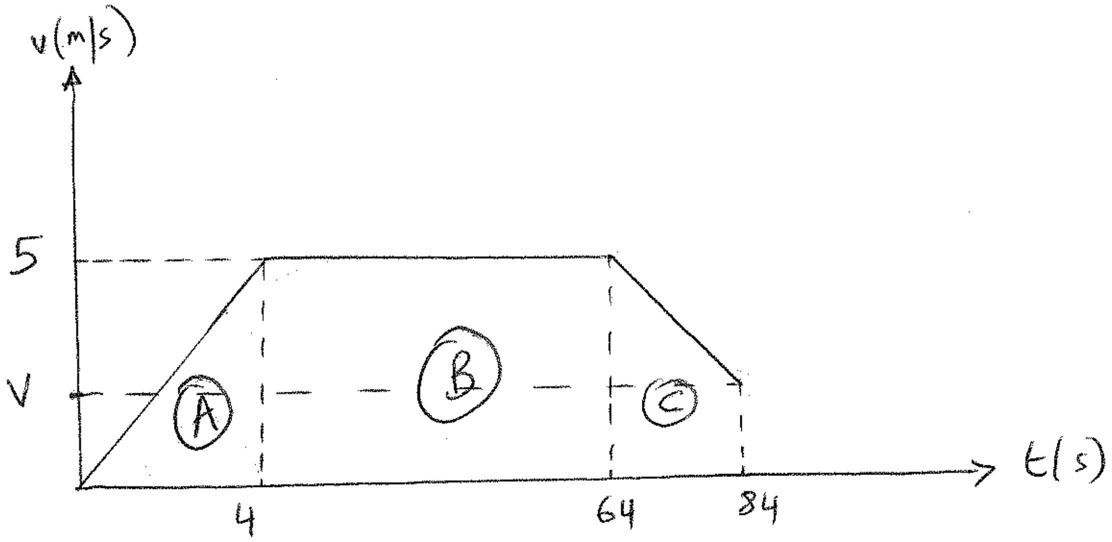
(a) Sketch, in the space below, a speed-time graph for the motion of the girl during the whole race. **(2)**

(b) Find the distance run by the girl in the first 64 s of the race. **(3)**

(c) Find the value of V. **(5)**

(d) Find the deceleration of the girl in the final 20 s of her race. **(2)**

a)



speed - time graph

b) Distance run by the girl in the first 64s of the race is simply the area of A and B

Area A $= \frac{5 \times 4}{2} = 10$ m.

Area B $= (64-4) \times 5 = 300$ m.

Distance run by the girl for the first 64 s is 310 m.

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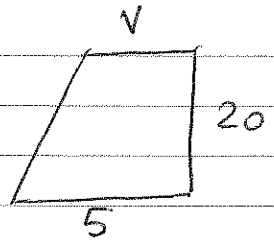
Question 4 continued



c) Let's consider shape C :

Area of C $= \left(\frac{v+5}{2}\right) \times 20$

$= 10(v+5)$. ①

On the other hand, we know that the girl's total distance is 400 m, she run 310 m for the first 64 s, which means that the distance left is 90 m :

① $\Rightarrow 10(v+5) = 90$

$\therefore v+5 = 9$

$\boxed{v = 4 \text{ m/s}}$

d) Deceleration = gradient of the line.

Let's Consider two points from this same line. (64, 5) and (84, 4).

$a = \text{gradient} = \frac{\text{Difference in } y}{\text{Difference in } x} = \frac{5-4}{64-84} = -\frac{1}{20}$

OR

$v = u + at \Rightarrow a = \frac{v-u}{t}$

$a = \frac{4-5}{20} = -\frac{1}{20}$

Deceleration is 0.05 m/s^2

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Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 12 marks)

P 3 8 1 6 1 R R A 0 1 3 2 4

Turn over

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5. A plank PQR, of length 8 m and mass 20 kg, is in equilibrium in a horizontal position on two supports at P and Q, where $PQ = 6$ m.

A child of mass 40 kg stands on the plank at a distance of 2 m from P and a block of mass M kg is placed on the plank at the end R. The plank remains horizontal and in equilibrium. The force exerted on the plank by the support at P is equal to the force exerted on the plank by the support at Q.

By modelling the plank as a uniform rod, and the child and the block as particles,

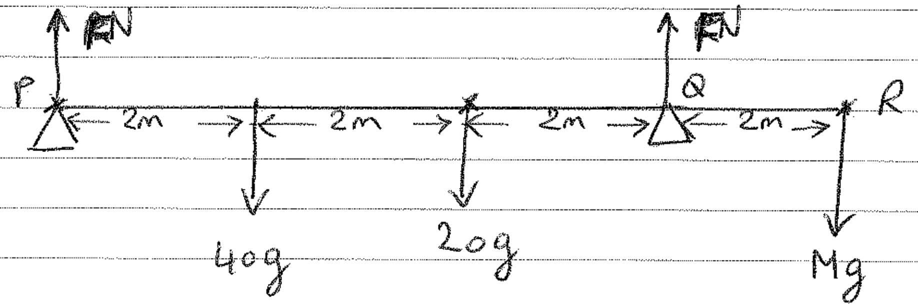
(a) (i) find the magnitude of the force exerted on the plank by the support at P,

(ii) find the value of M.

(10)

(b) State how, in your calculations, you have used the fact that the child and the block can be modelled as particles.

(1)



The plank is a uniform rod, the center of mass is in the middle of PR.

a) Plank in equilibrium $\Longleftrightarrow \begin{cases} \Sigma F = 0. \\ \Sigma M = 0 \end{cases}$

$\Sigma F = 0 : 2F = 40g + 20g + Mg.$

$\boxed{2F = 60g + Mg}$. (1)

P 3 8 1 6 1 R R A 0 1 4 2 4

Question 5 continued

$$\Sigma M_{/R} = 0 \ : \ F \times 2 + F \times 8 = 20g \times 4 + 40g \times 6$$

$$10F = 80g + 240g$$

$$10F = 320g$$

$$\boxed{F = 32g}$$

$$F = 313.6 \text{ N}$$

$$\boxed{F = 314 \text{ N}} \ (3sf)$$

(1) $2F = 60g + Mg$.

$$Mg = 2F - 60g.$$

$$Mg = 2 \times 32g - 60g$$

$$M = 64 - 60$$

$$\boxed{M = 4 \text{ kg}}$$

b) The child's mass and the block's mass were concentrated in one point only with the assumption that they were modelled as particles.

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Question 5 continued

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Question 5 continued

Q5

(Total 11 marks)

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6.

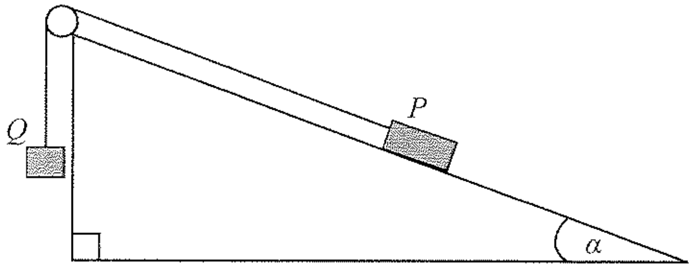


Figure 2

Two particles P and Q have masses 0.3 kg and m kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut, as shown in Figure 2.

The system is released from rest and Q accelerates vertically downwards at 1.4 m s^{-2}. Find

(a) the magnitude of the normal reaction of the inclined plane on P,

(2)

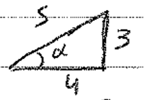
(b) the value of m.

(8)

When the particles have been moving for 0.5 s, the string breaks. Assuming that P does not reach the pulley,

(c) find the further time that elapses until P comes to instantaneous rest.

(6)



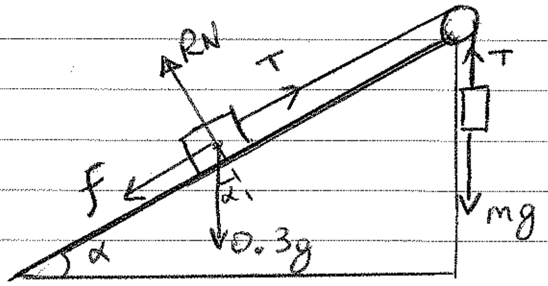
$\tan \alpha = \frac{3}{4}$

$\cos \alpha = \frac{4}{5}$

$\sin \alpha = \frac{3}{5}$

+

$\downarrow 1.4\ m/s^2$



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Question 6 continued

a) R?

Let's consider Particle P only.

Vertically, there is no mouvement taking place, sum of vertical components is nil.

$R = 0.3g\cos\alpha.$ $\quad R = 0.3 \times 9.8 \times \frac{4}{5}$

$\boxed{R = 2.35\,N}$

b) Value of m?

Particle P:

Solve forces parallel to the inclined plane,

Sum of forces = ma.

$-f - 0.3g\sin\alpha + T = 0.3 \times 1.4$ (*)

f being the friction force $f = \mu R$.

$+\mu \times 0.3g\cos\alpha + 0.3g\sin\alpha + 0.3 \times 1.4 = T$

$\boxed{T = 3.36\,N}$

Particle Q

~~The tension on the string is the same on both sides~~ of the pulley: "small smooth" pulley.

Leave blank

Question 6 continued

The inextensibility of the string means that both masses have the same acceleration.

The lighteness of the string means that the tension in it is constant.

$$-T + mg = ma.$$

$$m(g-a) = T$$

$$\boxed{m = \frac{T}{g-a}} \qquad m = \frac{3.36}{9.8-1.4}$$

$$\boxed{m = 0.4\,Kg}$$

c) Find the further time that elapses until P comes to rest.

Particle P

During the 0.5 sec : s

$u = 0\,m/s$ (P is held at rest)

$v = ?$

$a = 1.4\,m/s^2$

$t = 0.5\,sec$

$$v = u + at$$

$$= 0 + 1.4 \times 0.5$$

$$\underline{v = 0.7\,m/s}$$

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Question 6 continued

After $t = 0.5$ sec, once the string breaks:

s

$u = 0.7 m/s$.

$v = 0 m/s$ (until P comes to rest).

$a = ?$ new acceleration as the string breaks.

$t = ?$

Let's find the acceleration of the particle P when the string breaks $T = 0$.

Equation (*) becomes: $-f - 0.3g \sin\alpha = 0.3a$.

where a is the new acceleration when the string breaks.

$-\mu R - 0.3g \sin\alpha = 0.3a \Rightarrow$

$$a = \frac{-\frac{1}{2} \times 2.35 - 0.3 \times 9.8 \times \frac{3}{5}}{0.3} = -9.8 m/s^2$$

$$t = \frac{v - u}{a} = \frac{0 - 0.7}{-9.8} = 0.07 \text{ sec}$$

Q6

(Total 16 marks)

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7. *[In this question* **i** *and* **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.]*

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(b) Write down expressions, in terms of t, for

(i) $\mathbf{p}$,

(ii) $\mathbf{q}$,

(iii) $\overrightarrow{PQ}$.

(5)

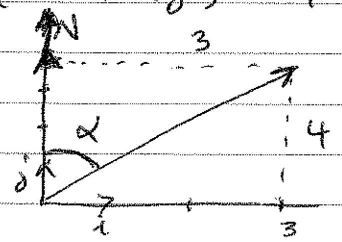
(c) Find the time when

(i) Q is due north of P,

(ii) Q is north-west of P.

(4)

a) Q moves with a velocity $(3i + 4j)$ km/h.



$\tan \alpha = \frac{3}{4}$

$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 37°$

The Bearing is 037°

b) i) $r = r_0 + vt$

$p = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} t$

$\boxed{p = (1+2t)i + (1-3t)j}$

Leave blank

Question 7 continued $r = r_0 + vt$

b ii) $q = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} t$

$\boxed{q = 3t\,i + (4t-2)\,j}$

b iii) $\overrightarrow{PQ} = q - p$

$= 3t\,i + (4t-2)\,j - [(1+2t)\,i + (1-3t)\,j]$

$\boxed{\overrightarrow{PQ} = (t-1)\,i + (7t-3)\,j}$

c) i) Q is due north of P if the i component of $\overrightarrow{PQ}$ is nil: $t - 1 = 0 \Rightarrow t = 1$.

<u>OR</u> Q is due north of P if the i component of p is equal to the i component of q.

$3t = 1 + 2t \Rightarrow t = 1$.

$t = 0 \Rightarrow 2\text{pm}$.

$t = 1 \Rightarrow \boxed{3\text{pm}}$

c ii) Q is due north-west of P if i component of $\overrightarrow{PQ}$ is equal and opposite to j component.

$t - 1 = -(7t-3)$

$8t = 4 \Rightarrow t = \frac{1}{2}$.

$t = 0 \Rightarrow 2\text{pm}$

$t = \frac{1}{2} \Rightarrow \boxed{2.30\text{pm}}$

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Question 7 continued

Q7

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

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