6. A bag contains a large number of balls.

65% are numbered 1

35% are numbered 2

A random sample of 3 balls is taken from the bag.

Find the sampling distribution for the range of the numbers on the 3 selected balls.

(6)

Possible samples:

$(1,1,1)$ range $= 0$ $\quad p = 0.65^3 = \frac{2197}{8000}$

$(1,1,2)\ (1,2,1)\ (2,1,1)$ range $= 1$

$p = 3(0.65)^2(0.35)$

$= \frac{3549}{8000}$

$(1,2,2)\ (2,1,2)\ (2,2,1)$ range $= 1$

$p = 3(0.65)(0.35)^2$

$= \frac{1911}{8000}$

$(2,2,2)$ range $= 0$ $\quad p = 0.35^3 = \frac{343}{8000}$

Sampling Dist

R	0	1
$P(R=r)$	$\frac{127}{400}$	$\frac{273}{400}$

7. The continuous random variable X has probability density function $\mathrm{f}(x)$ given by

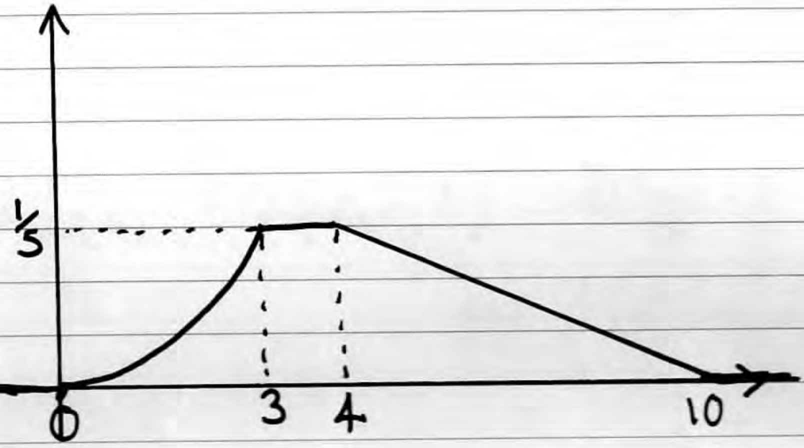
$$\mathrm{f}(x)=\begin{cases} \dfrac{x^2}{45} & 0 \leqslant x \leqslant 3 \\[2ex] \dfrac{1}{5} & 3 < x < 4 \\[2ex] \dfrac{1}{3}-\dfrac{x}{30} & 4 \leqslant x \leqslant 10 \\[2ex] 0 & \text{otherwise} \end{cases}.$$

(a) Sketch $\mathrm{f}(x)$ for $0 \leqslant x \leqslant 10$ **(4)**

(b) Find the cumulative distribution function $\mathrm{F}(x)$ for all values of x. **(8)**

(c) Find $\mathrm{P}(X \leqslant 8)$. **(2)**

a)



b) $\displaystyle\int \frac{x^2}{45}\,dx = \frac{x^3}{135} + C_1$

when $x=0$ $F(x)=0 \Rightarrow C_1 = 0$

$$\int \tfrac{1}{5}\,dx = \tfrac{x}{5} + C_2$$

when $x = 3$ $F(3) = \dfrac{3^3}{135} = \tfrac{1}{5}$

also $\tfrac{3}{5} + C_2 = \tfrac{1}{5} \Rightarrow C_2 = -\tfrac{2}{5}$

$$\int \left(\tfrac{1}{3} - \tfrac{x}{30}\right) dx = \tfrac{x}{3} - \tfrac{x^2}{60} + C_3$$

when $x = 10$ $F(10) = 1 \Rightarrow \tfrac{10}{3} + \dfrac{100}{60} + C_3 = 1$

$C_3 = -\tfrac{2}{3}$

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x^3}{135} & 0 \leq x \leq 3 \\ \dfrac{x}{5} - \dfrac{2}{5} & 3 < x < 4 \\ \dfrac{x}{3} - \dfrac{x^2}{60} - \dfrac{2}{3} & 4 \leq x \leq 10 \\ 1 & x > 10 \end{cases}$$

c) $P(X \leq 8) = F(8) = \dfrac{8}{3} - \dfrac{64}{60} - \dfrac{2}{3} = \underline{\underline{\dfrac{14}{15}}}$

8. In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.

A random sample of 10 customers is selected.

(a) Find the probability that

(i) exactly 6 ask for water with their meal,

(ii) less than 9 ask for water with their meal.

(5)

A second random sample of 50 customers is selected.

(b) Find the smallest value of n such that

$$P(X < n) \geqslant 0.9$$

where the random variable X represents the number of these customers who ask for water.

(3)

a) X = no. of customers asking for water

$X \sim B(10, 0.6)$

I) $P(X=6) = \binom{10}{6} 0.6^6 0.4^4 = \underline{\underline{0.2508}}$

II) Y = no. of customers not asking for water

$Y \sim B(10, 0.4)$

$P(X<9) = P(X \leq 8) = P(Y \geqslant 2)$

$= 1 - P(Y \leq 1)$

$= 1 - 0.0464$

$= \underline{\underline{0.9536}}$

[OR do $P(X=9) + P(X=10)$

$= 10(0.6)^9(0.4) + 0.6^{10}$

$= 0.04635$

$1 - \text{ANS} = 0.9536$]

Question 8 continued

b) $P(X<n) \geqslant 0.9$

$P(X \leq n-1) \geqslant 0.9$

$P(Y \geqslant 50-(n-1)) \geqslant 0.9$

$P(Y \geqslant 51-n) \geqslant 0.9$

$1 - P(Y \leq 50-n) \geqslant 0.9$

$0.1 \geqslant P(Y \leq 50-n)$

when $Y \sim B(50, 0.4)$ the first prob ≤ 0.1

is $x=15$ $p=0.0955$

$\therefore 50-n = 15$

$\underline{\underline{n = 35}}$