2. David claims that the weather forecasts produced by local radio are no better than those achieved by tossing a fair coin and predicting rain if a head is obtained or no rain if a tail is obtained. He records the weather for 30 randomly selected days. The local radio forecast is correct on 21 of these days.

Test David's claim at the 5% level of significance.

State your hypotheses clearly.

(7)

2. A traffic officer monitors the rate at which vehicles pass a fixed point on a motorway. When the rate exceeds 36 vehicles per minute he must switch on some speed restrictions to improve traffic flow.

(a) Suggest a suitable model to describe the number of vehicles passing the fixed point in a 15 s interval.

(1)

The traffic officer records 12 vehicles passing the fixed point in a 15 s interval.

(b) Stating your hypotheses clearly, and using a 5% level of significance, test whether or not the traffic officer has sufficient evidence to switch on the speed restrictions.

(6)

(c) Using a 5% level of significance, determine the smallest number of vehicles the traffic officer must observe in a 10 s interval in order to have sufficient evidence to switch on the speed restrictions.

(3)

2. A student takes a multiple choice test. The test is made up of 10 questions each with 5 possible answers. The student gets 4 questions correct. Her teacher claims she was guessing the answers. Using a one tailed test, at the 5% level of significance, test whether or not there is evidence to reject the teacher's claim.

State your hypotheses clearly.

(6)

5. A company has a large number of regular users logging onto its website. On average 4 users every hour fail to connect to the company's website at their first attempt.

(a) Explain why the Poisson distribution may be a suitable model in this case.

(1

Find the probability that, in a randomly chosen **2 hour** period,

(b) (i) all users connect at their first attempt,

(ii) at least 4 users fail to connect at their first attempt.

(5

The company suffered from a virus infecting its computer system. During this infection i was found that the number of users failing to connect at their first attempt, over a 12 hou period, was 60.

(c) Using a suitable approximation, test whether or not the mean number of users pe hour who failed to connect at their first attempt had increased. Use a 5% level o significance and state your hypotheses clearly.

(9

blank

6. A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample.

(2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025

(3)

(c) Find the actual significance level of this test.

(2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company's claim in the light of this value. Justify your answer.

(2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly.

(6)

6. (a) Define the critical region of a test statistic.

(2)

A discrete random variable X has a Binomial distribution $B(30, p)$. A single observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each tail which should be as close as possible to 0.005

(5)

(c) Write down the actual significance level of the test.

(1)

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region.

(2)

2. An effect of a certain disease is that a small number of the red blood cells are deformed. Emily has this disease and the deformed blood cells occur randomly at a rate of 2.5 per ml of her blood. Following a course of treatment, a random sample of 2 ml of Emily's blood is found to contain only 1 deformed red blood cell.

Stating your hypotheses clearly and using a 5% level of significance, test whether or not there has been a decrease in the number of deformed red blood cells in Emily's blood.

(6)

4. Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager questions whether or not there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(a) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's question. You should state the probability of rejection in each tail which should be less than 0.05.

(5)

(b) Write down the actual significance level of a test based on your critical region from part (a).

(1)

The manager found that 11 customers from the sample of 20 had bought baked beans in single tins.

(c) Comment on this finding in the light of your critical region found in part (a).

(2)

3. A single observation x is to be taken from a Binomial distribution $B(20, p)$.

This observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(a) Using a 5% level of significance, find the critical region for this test. The probability of rejecting either tail should be as close as possible to 2.5%.

(3)

(b) State the actual significance level of this test.

(2)

The actual value of x obtained is 3.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer.

(2)

6. A web server is visited on weekdays, at a rate of 7 visits per minute. In a random one minute on a Saturday the web server is visited 10 times.

(a) (i) Test, at the 10% level of significance, whether or not there is evidence that the rate of visits is greater on a Saturday than on weekdays. State your hypotheses clearly.

(ii) State the minimum number of visits required to obtain a significant result.

(7)

(b) State an assumption that has been made about the visits to the server.

(1)

In a random two minute period on a Saturday the web server is visited 20 times.

(c) Using a suitable approximation, test at the 10% level of significance, whether or not the rate of visits is greater on a Saturday.

(6)

3. A test statistic has a Poisson distribution with parameter λ.

Given that

$$\mathrm{H_0}: \lambda = 9, \ \mathrm{H_1}: \lambda \neq 9$$

(a) find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%.

(3)

(b) State the probability of incorrectly rejecting $\mathrm{H_0}$ using this critical region.

(2)

5. Sue throws a fair coin 15 times and records the number of times it shows a head.

(a) State the distribution to model the number of times the coin shows a head.

(2)

Find the probability that Sue records

(b) exactly 8 heads,

(2)

(c) at least 4 heads.

(2)

Sue has a different coin which she believes is biased in favour of heads. She throws the coin 15 times and obtains 13 heads.

(d) Test Sue's belief at the 1% level of significance. State your hypotheses clearly.

(6)