| Centre No. | | | | | | Paper Reference | | | | | | | Surname | Initial(s) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | MRS OSMANE | |
| Candidate No. | | | | | | 6 | 6 | 7 | 7 | / | 0 | 1 | Signature | |

Paper Reference(s)

# 6677/01

# Edexcel GCE

## Mechanics M1

## Advanced/Advanced Subsidiary

Friday 20 January 2012 – Afternoon

Time: 1 hour 30 minutes

HA008416413

| Materials required for examination | Items included with question papers |
|---|---|
| Mathematical Formulae (Pink) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.**

Examiner's use only

Team Leader's use only

| Question Number | Leave Blank |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| Total | |

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
Whenever a numerical value of $g$ is required, take $g = 9.8$ m s$^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.



Printer's Log. No.
P40096A
W850/R6677/57570 5/4/5/4/3

P40096A0128

*Turn over*

PEARSON

1. A railway truck $P$, of mass $m$ kg, is moving along a straight horizontal track with speed $15\,\mathrm{m\,s^{-1}}$. Truck $P$ collides with a truck $Q$ of mass 3000 kg, which is at rest on the same track. Immediately after the collision the speed of $P$ is $3\,\mathrm{m\,s^{-1}}$ and the speed of $Q$ is $9\,\mathrm{m\,s^{-1}}$. The direction of motion of $P$ is reversed by the collision.

Modelling the trucks as particles, find

(a) the magnitude of the impulse exerted by $P$ on $Q$,

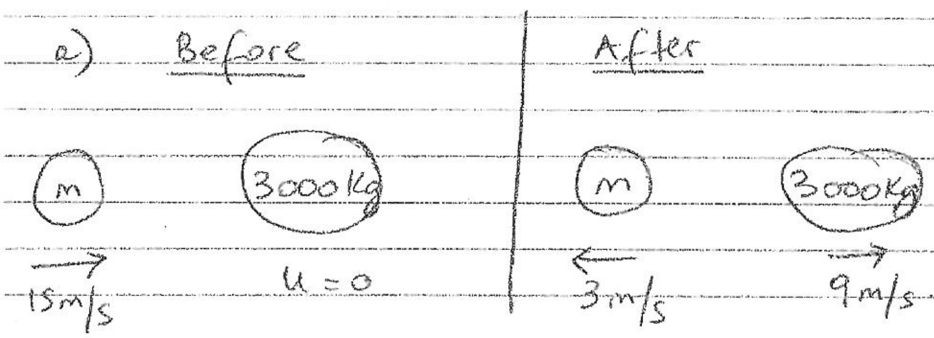
(2)

(b) the value of $m$.

(+) $\rightarrow$

(3)

a)




Principle of Impulse - Momentum:

$I = m(v - u)$

$= 3000(9 - 0)$

$\boxed{I = 27000 \text{ Ns}}$

b) Principle of Conservation of Momentum:

Total momentum before impact = Total momentum After impact.

$15m + 0 = -3m + 9 \times 3000$

$18m = 27000$

$\boxed{m = 1500 \text{ Kg}}$



P 4 0 0 9 6 A 0 2 2 8

Leave blank

**Question 1 continued**

Q1

**(Total 5 marks)**



P 4 0 0 9 6 A 0 3 2 8

2. A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and $R$ newtons respectively.

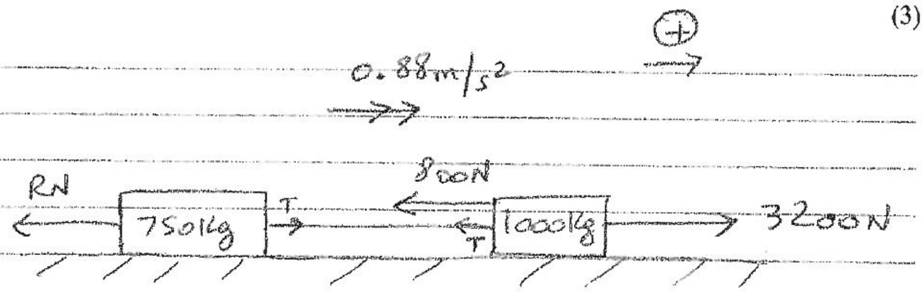
Given that the acceleration of the car and the caravan is $0.88\,\mathrm{m\,s^{-2}}$,

(a) show that $R = 860$,

(3)

(b) find the tension in the tow-bar.

(3)




Light rod ⇒ tension is cste.

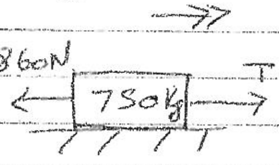
a) Let's consider the whole system : car + caravan.

Newton's second law of motion: $\Sigma F = ma$:

$-R - 800 + 3200 = (750 + 1000) \times 0.88$

$\boxed{R = 860\text{ N}}$

b) Let's consider the caravan only. 0.88m/s²




Newton's second law of motion: $\Sigma F = ma$.

$-860 + T = 750 \times 0.88$

$\boxed{T = 1520\text{ N}}$

Leave blank



P40096A0428

Leave blank

**Question 2 continued**

Q2

**(Total 6 marks)**



Turn over

Leave blank

**3.** Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle $P$ are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\text{ N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\text{ N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\text{ N}$$

where $p$ and $q$ are constants.

Given that $P$ is in equilibrium,

(a) find the value of $p$ and the value of $q$.

**(3)**

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.
Find

(b) the magnitude of $\mathbf{R}$,

**(2)**

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$.

**(3)**

$F_1\begin{pmatrix}7\\-9\end{pmatrix}$ ; $F_2\begin{pmatrix}5\\6\end{pmatrix}$ ; $F_3\begin{pmatrix}p\\q\end{pmatrix}$

a) P is in equilibrium : $\sum F = 0$

$F_1 + F_2 + F_3 = 0$ .

$\begin{pmatrix}7\\-9\end{pmatrix} + \begin{pmatrix}5\\6\end{pmatrix} + \begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow$

$\begin{cases}7+5+p=0\\-9+6+q=0\end{cases} \Rightarrow \begin{cases}p=-12\\q=3\end{cases}$

b) $R = F_1 + F_2 = \begin{pmatrix}7\\-9\end{pmatrix} + \begin{pmatrix}5\\6\end{pmatrix} = \begin{pmatrix}12\\-13\end{pmatrix}$

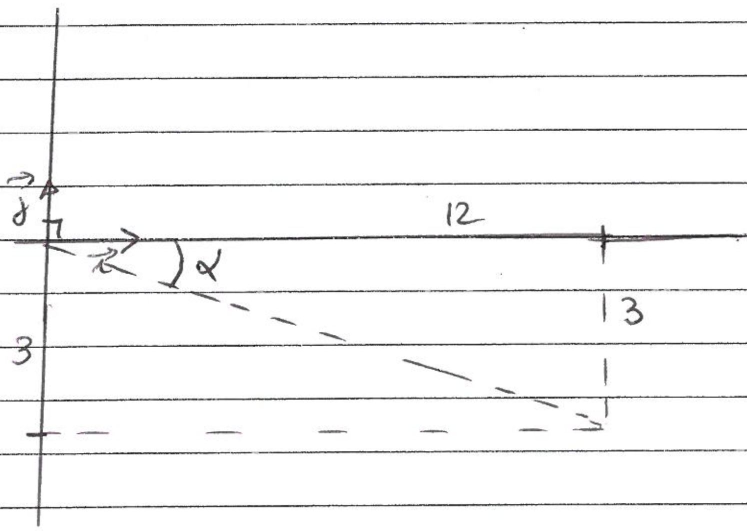
Magnitude of R : $|R| = \sqrt{12^2 + (-3)^2} = 12.4N$

Leave blank

**Question 3 continued**

c) $R = 12i - 3j$

R makes $(90° + \alpha)°$ with $j$




$\tan \alpha = \frac{3}{12} \Rightarrow \alpha = 14.036°$

Total angle $90 + 14.036 = 104°$ (3sf).

Q3

**(Total 8 marks)**

**4.**

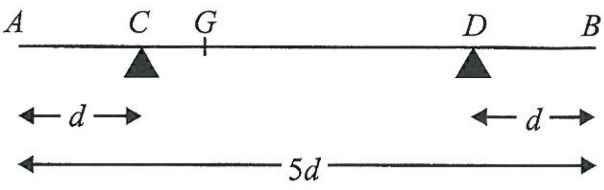



**Figure 1**

A non-uniform rod $AB$, of mass $m$ and length $5d$, rests horizontally in equilibrium on two supports at $C$ and $D$, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point $G$. A particle of mass $\frac{5}{2}m$ is placed on the rod at $B$ and the rod is on the point of tipping about $D$.

(a) Show that $GD = \frac{5}{2}d$.

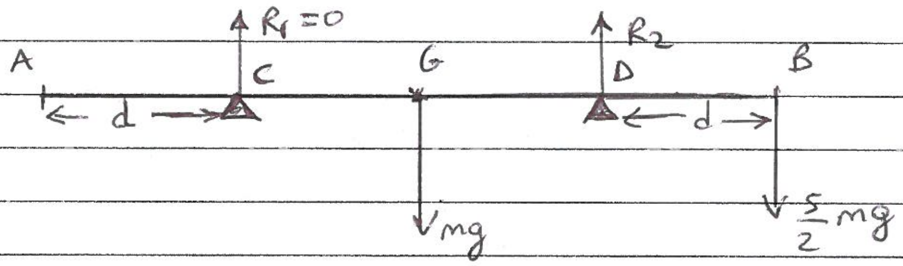
**(4)**

The particle is moved from $B$ to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at $D$ and the rod.

**(5)**

a)




$R_1 = 0$ : rod is on the point of tipping about D.

If the rod is about to tip about D $\Rightarrow \sum M_{/D} = 0$.

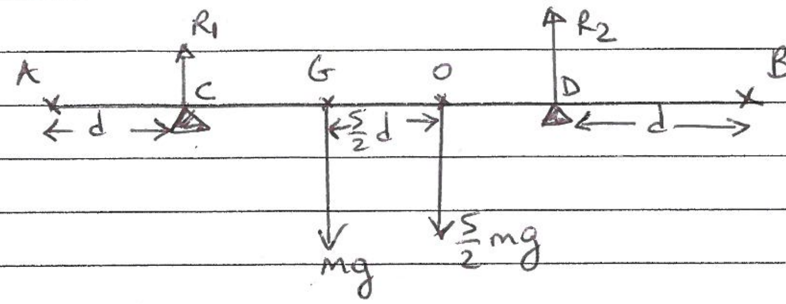
$mg \times GD = \frac{5}{2} mg \times d$

$\boxed{GD = \frac{5}{2} d}$

Leave blank

**Question 4 continued**

b)




$OA = OB = \frac{5}{2}d$ (O is midpoint of AB).

$CG = CD - GD$ ; $OC = \frac{5}{2}d - d$

$CG = 3d - \frac{5}{2}d$. $OC = \frac{3}{2}$

$CG = \frac{d}{2}$

Rod in equilibrium $\Longrightarrow \sum M_{/C} = 0$.

$mg \times CG + \frac{5}{2}mg \times OC = R_2\, CD$.

$mg \times \frac{d}{2} + \frac{5}{2}mg \times \frac{3}{2}d = R_2 \times 3d$

$\frac{1}{2}mg + \frac{15}{4}mg = 3R_2$

$\frac{17}{4}mg = 3R_2$

$\boxed{R_2 = \frac{17}{12}mg}$



Turn over

Leave blank

**Question 4 continued**

Leave blank

**Question 4 continued**

Q4

**(Total 9 marks)**

Leave blank

5. A stone is projected vertically upwards from a point $A$ with speed $u$ m s$^{-1}$. After projection the stone moves freely under gravity until it returns to $A$. The time between the instant that the stone is projected and the instant that it returns to $A$ is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$,

**(3)**

(b) find the greatest height above $A$ reached by the stone,

**(2)**

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above $A$.

**(6)**

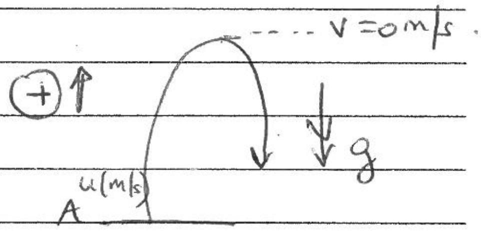
a) $s = 0$

$u$ ?

$v$

$a = -9.8 \text{ m/s}^2$

$t = 3\frac{4}{7}$ sec




$s = ut + \frac{1}{2}at^2$

$0 = u \times 3\frac{4}{7} + \frac{1}{2}(-9.8)(3\frac{4}{7})^2$

$u = \frac{1}{2} \times 9.8 \times 3\frac{4}{7}$

$\boxed{u = 17\frac{1}{2} \text{ m/s}}$

b) Greatest height above A?

$s = ?$

$u = 17.5$ m/s

$v = 0$ m/s (stone changes direction at max height)

$a = -9.8 \text{ m/s}^2$

$t$

$v^2 - u^2 = 2as \Rightarrow s = \frac{v^2 - u^2}{2a} = \frac{0 - 17.5^2}{-2 \times 9.8}$

$\boxed{s = 15.6 \text{ m}}$

Leave blank

**Question 5 continued**

c) $s = 6\frac{3}{5} = 6.6\text{m}.$

$u = 17.5\text{m/s}.$

$v$

$a = -9.8\text{m/s}^2$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$6.6 = 17.5t - 4.9t^2 \Rightarrow 4.9t^2 - 175t + 66 = 0.$

This is a quadractic equation of the format

$ax^2 + bx + c = 0 \qquad x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

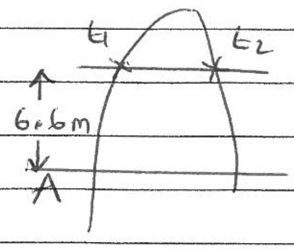
$x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$t_1 = \frac{175 - \sqrt{175^2 - 4 \times 4.9 \times 66}}{2 \times 4.9} = \frac{42}{98} = \frac{3}{7}$ sec

$t_2 = \frac{175 + \sqrt{175^2 - 4 \times 4.9 \times 66}}{2 \times 4.9} = \frac{308}{98} = \frac{22}{7}$ sec

The stone is $6\frac{3}{5}$ m above A, $\frac{3}{7}$ sec, on the way up,

and $\frac{22}{7}$ sec on the way down




The Length of time for which the

stone was at least $6\frac{3}{5}$ m above A

is $\frac{22}{7} - \frac{3}{7} = \frac{19}{7}$ sec.

Leave blank

**Question 5 continued**

Leave blank

**Question 5 continued**

Q5

**(Total 11 marks)**

Leave blank

**6.** A car moves along a straight horizontal road from a point $A$ to a point $B$, where $AB = 885$ m. The car accelerates from rest at $A$ to a speed of $15\,\text{m}\,\text{s}^{-1}$ at a constant rate $a\,\text{m}\,\text{s}^{-2}$. The time for which the car accelerates is $\frac{1}{3}T$ seconds. The car maintains the speed of $15\,\text{m}\,\text{s}^{-1}$ for $T$ seconds. The car then decelerates at a constant rate of $2.5\,\text{m}\,\text{s}^{-2}$ stopping at $B$.

(a) Find the time for which the car decelerates. **(2)**

(b) Sketch a speed-time graph for the motion of the car. **(2)**

(c) Find the value of $T$. **(4)**

(d) Find the value of $a$. **(2)**

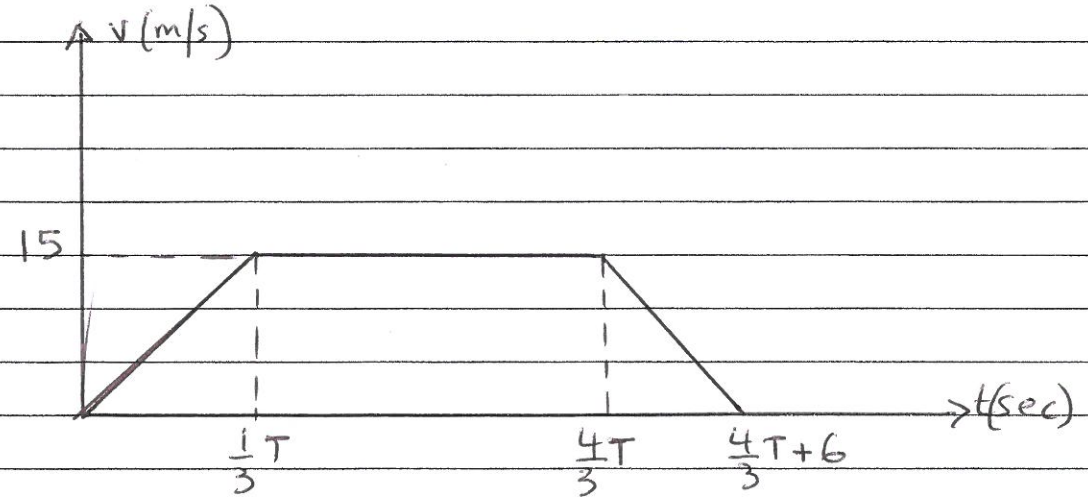
(e) Sketch an acceleration-time graph for the motion of the car. **(3)**

a) s.
$u = 15$ m/s
$v = 0$ m/s (stopping at B)
$a = -2.5$ m/s²
$t = ?$

$v = u + at \Rightarrow t = \dfrac{v-u}{a}$

$t = \dfrac{-1.5}{-2.5}$ $\boxed{t = 6 \text{ sec}}$

b)




speed-time graph.

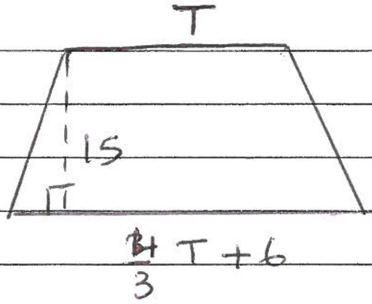
Leave blank

**Question 6 continued**

c) T? , Distance = 885 m.

Total distance = area under the graph.

The area of a trapezium is $A = \frac{a+b}{2} \times h$



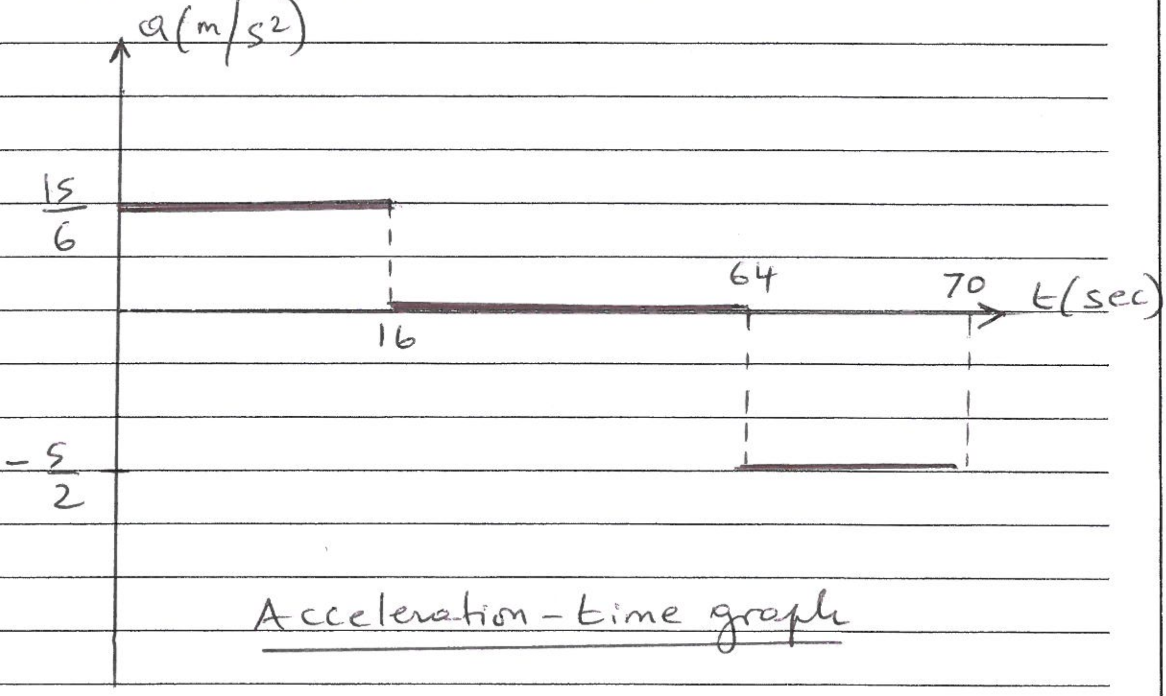

$$885 = \frac{T + \frac{4}{3}T + 6}{2} \times 15.$$

$7T = 112 \Rightarrow \boxed{T = 48 \text{ sec}}$

d) Let a be the gradient of the first phase of the speed-time graph.

$a = \frac{15}{\frac{1}{3}T} = \frac{15}{16}$, $\boxed{a = 0.94 \, m/s^2}$

e)




Acceleration-time graph

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

Q6

**(Total 13 marks)**

Leave blank

7. [*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.*]

A boat $P$ is moving with constant velocity $(-4\mathbf{i}+8\mathbf{j})$ km h$^{-1}$.

(a) Calculate the speed of $P$.

**(2)**

When $t = 0$, the boat $P$ has position vector $(2\mathbf{i}-8\mathbf{j})$ km. At time $t$ hours, the position vector of $P$ is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of $t$.

**(1)**

A second boat $Q$ is also moving with constant velocity. At time $t$ hours, the position vector of $Q$ is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i}+8\mathbf{j})$$

Find

(c) the value of $t$ when $P$ is due west of $Q$,

**(3)**

(d) the distance between $P$ and $Q$ when $P$ is due west of $Q$.

**(3)**

a) $\vec{v} = -4i + 8j$

$|v| = \sqrt{(-4)^2 + 8^2}$

$\boxed{|v| = 8.94 \text{ km/h}}$ (3sf)

b) $t=0$ position $\vec{p}_0 = 2i - 8j$

At $t$: $\vec{p} = \vec{p}_0 + \vec{v}t$

$\vec{p} = (2i - 8j) + (-4i + 8j)t$

$\boxed{\vec{p} = (2-4t)\vec{i} + (8t-8)\vec{j}}$

**Question 7 continued**

c) $\vec{p} = (2-4t)\vec{i} + (8t-8)\vec{j}$

$\vec{q} = 18\vec{i} + 12\vec{j} + t(6i + 6j)$

$\underline{q = (18-6t)\vec{i} + (12-8t)\vec{j}}$

P is due west of Q $\Rightarrow$ equal j component.

$8t - 8 = 12 - 8t$

$16t = 20 \Rightarrow \boxed{t = 1.25h}$

d) Distance between P and Q at $t = 1.25h$.

$\vec{P} = -3\vec{i} + 2\vec{j}$

$\vec{q} = 10.5\vec{i} + 2\vec{j}$

$\vec{PQ} = \vec{q} - \vec{p} = (10.5 - -3)\vec{i} + (2-2)\vec{j}$

$= 13.5\vec{i}$

$|PQ| = \sqrt{13.5^2}$

$\boxed{|PQ| = 13.5km}$

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

Q7

**(Total 9 marks)**

Leave blank

**8.**

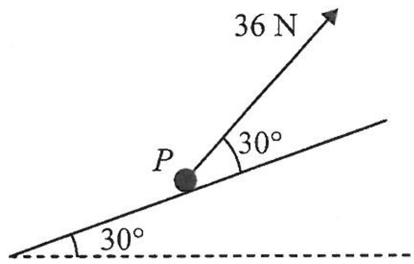



**Figure 2**

A particle $P$ of mass 4 kg is moving up a fixed rough plane at a constant speed of $16\,\text{m}\,\text{s}^{-1}$ under the action of a force of magnitude 36 N. The plane is inclined at 30° to the horizontal. The force acts in the vertical plane containing the line of greatest slope of the plane through $P$, and acts at 30° to the inclined plane, as shown in Figure 2. The coefficient of friction between $P$ and the plane is $\mu$. Find

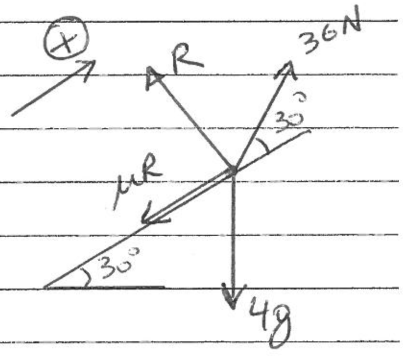
(a) the magnitude of the normal reaction between $P$ and the plane, **(4)**

(b) the value of $\mu$. **(5)**

The force of magnitude 36 N is removed.

(c) Find the distance that $P$ travels between the instant when the force is removed and the instant when it comes to rest. **(5)**

a) P is moving at a constant speed $\Rightarrow a = 0\,m/s^2$




Resolve forces:

(↑) vertically:

$R + 36\sin 30 = 4g\cos 30°$

$R = 4g\cos 30° - 36\sin 30°$

$\boxed{R = 15.9\,N}$ (3sf)

**Question 8 continued**

($\rightarrow$) Horizontally:

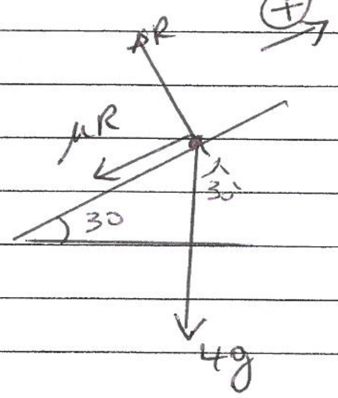
$$\mu R + 4g \sin 30 = 36 \cos 30^\circ$$

$$\mu = \frac{36 \cos 30^\circ - 4g \sin 30^\circ}{R}$$

$$\boxed{\mu = 0.73}$$

c) The 36N force is removed but the particle will continue moving up until it stops (rest).

Let's resolve forces // and ⊥ to the inclined plane.




($\uparrow$) vertically:

$$R = 4g \cos 30^\circ$$

($\rightarrow$) Horizontally: $-4g \sin 30 - \mu R = 4a$.

$$a = \frac{-4g \sin 30^\circ - 0.73 \times (4g \cos 30)}{4} = -11.09$$

Deceleration is $11.09 \ m/s^2$.

$$v^2 - u^2 = 2as \Rightarrow s = \frac{-u^2}{2a}$$

$\begin{cases} v = 0 : \text{P comes to rest} \\ u = 16 \ m/s \\ a = -11.09 \ m/s^2 \end{cases}$

$\boxed{s = 11.6 \ m}$ (3sf)

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

Leave blank

**Question 8 continued**

Q8

**(Total 14 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**